Relationship between density, pressure, and temperature

- What happens to density if pressure increases?
  - Increases \( \rho \sim P \)  
    (Boyle’s Law)

- What happens to density if temperature increases?
  - Decreases \( \rho \sim 1/T \)  
    (Charle’s Law)

- What happens to pressure if temperature increases?
  - Increases \( P \sim T \)  
    (Gay-Lussac’s Law)

- Putting all 3 together, we have \( P \sim \rho \times T \)
Relationship between density, pressure, and temperature

• The ideal gas law for dry air

\[ P = \rho R_d T \]  

– \( R_d \): gas constant for dry air
  • Equals to 287 J/kg/K
  – Note that P, \( \rho \), and T have to be in S.I. units for this equation to work using this value of \( R_d \)
Numerical example

• What is the pressure of dry air with a temperature of 10 °C and a density of 1 kg/m³?
  – Use the ideal gas law: \( P = \rho R_d T \)
  – Need to express all quantities in S.I. units
    • \( T = 10 \, \text{°C} = 10 + 273.15 \, \text{K} = 283.15 \, \text{K} \)
    • \( \rho = 1 \, \text{kg/m}^3 \) already in S.I. units
  – \( p = \rho R_d T = 1 \times 287 \times 283.15 = 81264 \, \text{Pa} \)
Classwork example

• What is the density of dry air with a temperature of 20 °C and a pressure of 800 hPa?
  – Use the ideal gas law: \( P = \rho R_d T \)
  – Need to express all quantities in S.I. units
    • \( T = 20 \, ^\circ\text{C} = 20 + 273.15 \, \text{K} = 293.15 \, \text{K} \)
    • \( P = 800 \, \text{hPa} = 800 \times 100 \, \text{Pa} = 80000 \, \text{Pa} \)
    – \( \rho = P/R_d/T = 80000/287/293.15 = 0.95 \, \text{kg/m}^3 \)
• For moist air:
  – Question for thought: At the same temperature and pressure, is moist air more dense or less dense than dry air?

  – Answer: Less dense!
    • At the same temperature and pressure, the same volume of gas contains the same number of molecules. Since \( \text{H}_2\text{O} \) molecules weigh less than \( \text{N}_2 \) or \( \text{O}_2 \) molecules, the same volume of moist air actually weighs less than the same volume of dry air.
• Ideal gas law for moist air:
  – One way to allow for the difference between moist and dry air is by using a different gas constant for moist air
  – However, meteorologists chose instead to define a “virtual temperature”
    \[ T_v = T \cdot (1 + 0.61r) \]  \hspace{1cm} \text{Stull (1.13)}
    \[ P = \rho R_d T_v \]  \hspace{1cm} \text{Stull (1.15)}
  • \( r \) is water vapor mixing ratio
  – In a nutshell, moist air of temperature \( T \) behaves as dry air with temperature \( T_v \)
Review of equations covered in Chapter 1

- **Density:** $\rho = \frac{M}{V}$ (mass per unit volume)
- **Pressure:** $P = \frac{F}{A}$ (force per unit area)
- **Hydrostatic equation:** $\Delta \rho = -\rho g \Delta z$
- **Transformation of temperature**
  - $T_K = T_C + 273.15$  \quad $T_F = T_C \times 9/5 + 32$
- **Ideal gas law**
  - For dry air: $P = \rho R_d T$  \quad $R_d = 287 \text{ J/kg/K}$
  - For moist air: $P = \rho R_d T_v$
    - Where $T_v = T (1 + 0.61 r)$ (virtual temperature)
      - $r = \text{mass of water vapor/mass of dry air}$
        - (water vapor mixing ratio)
- **Note that unless specified, most equations only work if S.I. units are used**