Relationship between density, pressure, and temperature

- What happens to density if pressure increases?
  - Increases $\rho \sim P$ (Boyle's Law)

- What happens to density if temperature increases?
  - Decreases $\rho \sim 1/T$ (Charle's Law)

- What happens to pressure if temperature increases?
  - Increases $P \sim T$ (Gay-Lussac's Law)

- Putting all 3 together, we have $P \sim \rho \times T$

Relationship between density, pressure, and temperature

- The ideal gas law for <u>dry air</u>

$$P = \rho R_d T$$

Stull (1.12)

  - R_d: gas constant for dry air
    - Equals to 287 J/kg/K
  - Note that P, ρ, and T have to be in S.I. units for this equation to work using this value of R_d

Numerical example

- What is the pressure of dry air with a temperature of 10 ^oC and a density of 1kg/m^3?
  - Use the ideal gas law: $P = \rho R_d T$
  - Need to express all quantities in S.I. units
    - T = 10 ^oC = 10+273.15 K = 283.15 K
    - ρ = 1 kg/m^3 already in S.I. units
  - $p = \rho R_d T$ = 1x287x283.15 = 81264 Pa

Classwork example

- What is the density of dry air with a temperature of 20 °C and a pressure of 800 hPa?
  - Use the ideal gas law: $P = \rho R_d T$
  - Need to express all quantities in S.I. units
    - T = 20 °C = 20+273.15 K = 293.15 K
    - P = 800 hPa = 800 x 100 Pa = 80000 Pa
  - $\rho = P/R_d/T = 80000/287/293.15 = 0.95\ kg/m^3$

- For moist air:
  - Question for thought: At the same temperature and pressure, is moist air more dense or less dense than dry air?

  - Answer: Less dense!
    - At the same temperature and pressure, the same volume of gas contains the same number of molecules. Since H_2O molecules weigh less than N_2 or O_2 molecules, the same volume of moist air actually weighs less than the same volume of dry air

- Ideal gas law for moist air:
  - One way to allow for the difference between moist and dry air is by using a different gas constant for moist air
  - However, meteorologists chose instead to define a “virtual temperature”

$$T_v = T \cdot (1 + 0.61r)$$

Stull (1.13)

    - *r* is water vapor mixing ratio
  - Ideal gas law for moist air:

$$P = \rho R_d T_v$$

Stull (1.15)

    - In a nutshell, moist air of temperature *T* behaves as dry air with temperature T_v

Review of equations covered in Chapter 1

- Density: $\rho = M/V$ (mass per unit volume)
- Pressure: $P = F/A$ (force per unit area)
- Hydrostatic equation: $\Delta p = -\rho\, g\, \Delta z$
- Transformation of temperature
  - $T_K = T_C + 273.15$ $T_F = T_C \times 9/5 + 32$
- Ideal gas law
  - For dry air: $P = \rho R_d T$ $R_d = 287\ J/kg/K$
  - For moist air: $P = \rho R_d T_v$
    - Where $T_v = T\,(1 + 0.61\,r)$ (virtual temperature)
      - r = mass of water vapor/mass of dry air (water vapor mixing ratio)
- Note that unless specified, most equations only work if S.I. units are used